Mg-Stearate. The stearate coated the PVC pellets forming sort of a "semi-sticky" powder, which tended to stick to a curved surface and gather at the nodes of a vibrating shell. The powder proved to be quite successful, and some rather exotic nodal patterns were detected. Because of the poor photographic quality, pictures showing the most interesting patterns detected in the study have not previously been placed in the open literature. However, in view of possible general interest, these are shown in Figs. 2 and 3. In Fig. 2, the mode consisting of $4\frac{1}{2}$ axial waves and 15 circumferential waves was observed at 3500 cps. Figure 3 shows the mode (4-19) at its natural frequency of 4500 cps.

References

[1] Watkins, J. D. and Clary, R. R., "Vibrational characteristics of thin-walled conical frustum shells," AIAA J. **2**, 1815–1816 (1964).

[2] Hu, W. C. L., "Comments on 'Vibrational characteristics of thin-walled conical frustum shells,' " AIAA J. **3**, 1213 (1965).

[3] Watkins, J. D. and Clary, R. R., "Reply by authors to W. C. L. Hu," AIAA J. **3**, 1213–1214 (1965).

[4] Tobias, S. A., "A theory of imperfection for the vibration of elastic bodies of revolution," Eng. **172**, 409 (1951).

[5] Arnold, R. N. and Warburton, G. B., "Flexural vibrations of the walls of thin cylindrical shells having freely supported engs," Proc. Roy. Soc. (London) **A197**, 238–257 (1949).

[6] Koval, L. R. and Cranch, E. T., "On the free vibrations of a thin-walled cylindrical shell subjected to an initial static torque," *Proceedings of the Fourth U. S. National Congress of Applied Mechanics* (University of California Press, Berkeley, Calif., 1962), pp. 18–21.

[7] Koval, L. R., "Note on effect of dynamic asymmetry on the vibration of cylindrical shells," J. Acoust. Soc. Am. **35**, 252–253 (1963).

Comments on Similarity Solutions of the Boundary Layer Near an Accelerating Plate

Arthur N. Tifford*
P. O. Box 611, Encino, Calif.

THIS comment was conceived in part as a critical response to the publication of similar title by Na[1] and in part as an action of support for the "Plea, and Clarion Call" of the Editor.[2] Its essence is that Na's claim to a generalization of such classic works of more than fifty and a hundred years ago as those of Blasius[3] and Stokes[4] is patently nothing more than just another case of "claim-jumping." The literature of the intervening years fully document the priority of earlier relevant works of others.

The first study of similar boundary-layer solutions as a special class goes back some 35 years to the pioneering efforts of Falkner and Skan.[5] Subsequent, relatively well-known papers on the subject were written by Goldstein,[6] Mangler,[7] and Thwaites.[8] The adaptation of group ideas to the solution of this boundary value problem goes back at least 15 years.[9]

The early similar boundary-layer research was concerned primarily with steady-state flows. But what of unsteady similar boundary-layer studies? Comprehensive treatments of this subject go back at least ten years to Schuh[10] (referenced in the fourth edition of Schlichting[11]). In fact, among Schuh's findings are the particular power law and exponential time variations of the relative plate velocity displayed by Na.

Received March 29, 1965.

* Research Consultant. Associate Fellow Member AIAA.

But one might object that all of the referenced papers deal exclusively with Newtonian fluids. To such an objection there are at least two direct replies.

1) Relatively elementary analysis shows that the use of the so-called Ostwald-de Waele power law fluid model instead of the classical Newtonian one alters the friction term in Prandtl's boundary-layer equations in no essential way as far as the determination of the relative inviscid velocity fields allowing similar boundary-layer profiles is concerned. All fluid models with an effective viscosity representation as in Eq. (1) have this common characteristic:

$$\mu = a\,|\partial^r u/\partial y^r|^{n-1} \tag{1}$$

More general viscosity "laws" with this special quality pertain in particular cases. For example, when the relative inviscid velocity is constant, except for a possible step function change initially, similarity tolerates the more general viscosity relation of Eq. (2):

$$\mu = \sum_r a_r \left|\frac{\partial^r u}{\partial y^r}\right|^{(n-1)/r} \tag{2}$$

Then again, when the similarity independent variable is only a function of the normal distance from the surface, as in a forward stagnation flow, the effective viscosity is allowed to be of the general form of Eq. (3):

$$\mu = \sum_r a_r \left|\frac{\partial^r u}{\partial y^r}\right|^{n-1} \tag{3}$$

2) Quite apart from the above, reference to the more recent literature reveals that similar boundary-layer solutions with the power law fluid model have already been spelled out by others for a wide range of circumstances, including the accelerating plate configuration. In fact Ref. 12, from which Na's Ref. 6[13] was extracted, contains the results he claims.

References

[1] Na, T. Y., "Similarity solutions of the flow of power law fluids near an accelerating plate, AIAA J. **3**, 378 (1965).

[2] Steg, L., "An appreciation, a plea, and a clarion call," AIAA J. **3**, 1–2 (1965).

[3] Blasius, H., "Grenzschichten in Flussigkeiten mit kleiner Reiburg," Z. Math. Phys. **56**, 1–37 (1908).

[4] Stokes, G. G., "On the effect of the internal friction of fluids on the motion of pendulums," Cambr. Phil. Trans. **IX**, 8–106 (1851).

[5] Falkner, V. M. and Skan, S. W., "Some approximate solutions of the boundary layer equations," British Air Ministry, Aeronautical Research Committee R & M 1314 (April 1930).

[6] Goldstein, S., "A note on the boundary layer equations," Proc. Cambr. Phil. Soc. **35**, 388 (1939).

[7] Mangler, E., "Die 'ahnlichen' Losungen der Prandtlschen Grenzschichtgleichunger," Z. Angew. Math. Mech. **23**, 241–251 (1943).

[8] Thwaites, B., "On certain types of boundary-layer flow with continuous surface suction," British Air Ministry, Aeronautical Research Committee R & M 2243 (July 1946).

[9] Tifford, A. N., "On certain particular solutions of the laminar boundary-layer equations," *Proceedings of the 1950 Midwestern Conference on Fluid Dynamics* (J. W. Edwards, Ann Arbor, Mich., 1951).

[10] Schuh, H., "Uber die 'ahnlichen' Losungen der instationaren laminaren Grenzschichtgleichung in inkompressibler Stromung," *50 Jahre Grenzschichtforschung* (Friedrich Vieweg & Sohn, Braunschweig, Germany, 1955), pp. 147–152.

[11] Schlichting, H., *Boundary Layer Theory* (McGraw-Hill Book Co., Inc., New York, 1960), 4th ed., p. 134.

[12] Wells, C. S., "Similar solutions of the boundary layer equations for purely viscous non-Newtonian fluids," NASA TN-D-2262 (April 1964).

[13] Wells, C. S., "Unsteady boundary-layer flow of a non-Newtonian fluid on a flat plate," AIAA J. **2**, 951–952 (1964).